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NONEQUILIBRIUM EXPANSION OF A PLASMA FROM A THERMIONIC SOURCE

by Peter M. Sockol

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1963

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SUMMARY

The nonequilibrium expansion of a partially ionized gas through a convergent-divergent nozzle is analyzed with a three-body recombination coefficient. The results of calculations for alkali metal plasmas, such as might be produced in a thermionic diode, are presented. They indicate that a large percentage of the ionization energy can be recovered before freezing becomes dominant. In particular, for lithium initially 60 percent ionized at 10 millimeters of mercury, 0.789 of the available thrust and a specific impulse of 960 seconds were obtained.

INTRODUCTION

At the present time, there is a fair amount of evidence to support the thesis that highly elevated electron temperatures can be produced in a plasma diode. It has been theorized (ref. 1) that, if the energy gained by electrons when they are accelerated through an ion-rich emitter sheath is randomized, perhaps by plasma oscillations, a high-temperature electron gas will result. The experimental confirmations include Langmuir probe (ref. 2) and spectroscopic measurements (refs. 3 and 4). The presence of a high electron temperature indicates that a considerable amount of energy has been stored in the plasma in the form of ionization energy and electron thermal energy. In this report, an analysis is made of the feasibility of the conversion of this energy into thrust by expansion of the plasma through a nozzle.

The system under consideration is one in which an alkali-metal vapor is allowed to flow through a thermionic diode and then to expand through a convergent-divergent nozzle. The diode would be externally short circuited to transfer the maximum amount of energy into the plasma. There are three major problems of interest in the analysis of this system. The first involves the prediction of electron temperature and density inside the diode from a knowledge of such parameters as surface temperatures and gas pressure. The second concerns the relaxation of the plasma from a highly nonequilibrium state within the diode to some state much closer to equilibrium at the entrance to the nozzle. Both of these problems are beyond the scope of this report. The third problem deals with the recovery of the energy stored in the plasma. Because of the finite recombination rates, it is expected that a certain amount of chemical freezing will occur at the higher flow velocities. This will put a limit on the amount of ionization energy that can be recovered in the form of thrust. The expansion problem with finite recombination rates is analyzed herein.

ANALYSIS

The analysis is made within the framework of the following assumptions: the flow is quasi-one-dimensional, adiabatic, and steady; the plasma can be idealized as a mixture of three perfect gases with constant specific heats (electrons, ions, and monatomic neutral atoms); and the electron temperature is equal to the gas temperature. With these assumptions, the equations of conservation of energy, momentum, and mass are as follows (ref. 5):

$$h + \frac{1}{2} v^2 = h_0 \tag{1}$$

$$\rho v \frac{dv}{dx} = - \frac{dP}{dx} \tag{2}$$

$$\rho v a = W \tag{3}$$

(Symbols are defined in appendix A.) The equations of state are (ref. 6)

$$P = \rho \frac{k}{m_a} T(1 + f) \tag{4}$$

$$h = 2.5 \frac{k}{m_a} T(1 + f) + \frac{E}{m_a} f \tag{5}$$

where m_a is the atomic mass, f the degree of ionization, and E the ionization energy per atom. At the temperatures to be considered, the specific heats are not constant, but the second term in equation (5) should mask this effect.

In addition to equations (1) to (5) a chemical rate equation for the ionization reaction is needed. If R is the net rate of increase of electrons per unit volume per unit time, then, for the collisional reaction

$$A^o + e^- \rightleftarrows A^+ + 2e^- \tag{6}$$

the reaction rate R_c is

$$R_c = S_{ic}(T) n_a n_e - S_{rc}(T) n_e^3$$

where $S_{ic}(T)$ and $S_{rc}(T)$ are the rate constants for ionization and recombination, respectively; n_a and n_e are the neutral-atom and electron densities, respectively; and the ion density has been set equal to the electron density. Since S_{ic} and S_{rc} are related through the equilibrium constant K_{eq}:

$$S_{ic} = S_{rc}K_{eq}$$

it follows that

$$R_c = -S_{rc}n_e(n_e^2 - K_{eq}n_a) = -\alpha_c(n_e^2 - K_{eq}n_a) \tag{7}$$

For the radiative reaction

$$A^o \rightleftarrows A^+ + e^- \tag{8}$$

the reaction rate R_r is

$$R_r = S_{ir}(T)n_a - S_{rr}(T)n_e^2$$

$$= -S_{rr}(n_e^2 - K_{eq}n_a) = -\alpha_r(n_e^2 - K_{eq}n_a) \tag{9}$$

The net reaction rate is taken to be

$$R = R_c + R_r = -\alpha(n_e^2 - K_{eq}n_a) \tag{10}$$

For the recombination coefficient α, the value given in reference 7 is used:

$$\alpha = \alpha_c + \alpha_r \approx 5.6\times10^{-27}(kT)^{-4.5}\, n_e + 2.7\times10^{-13}(kT)^{-0.75} \qquad (cm^3)(sec^{-1}) \tag{11}$$

where kT is in electron volts. In differential form, the steady-state continuity equations for electrons and mass are

$$\frac{d}{dx}(n_e va) = Ra \tag{12}$$

$$\frac{d}{dx}(\rho va) = 0 \tag{13}$$

Elimination of a in these equations gives

$$\rho v \frac{d}{dx}(n_e/\rho) = R \tag{14}$$

The neutral-atom and electron densities are

$$n_e = f\rho/m_a \tag{15}$$

$$n_a = (1 - f)\rho/m_a \tag{16}$$

Substitution of these relations into equations (10) and (14) yields

$$v\frac{df}{dx} = -\alpha \frac{\rho}{m_a}\left[f^2 - G(1 - f^2)\right] \tag{17}$$

The parameter G is given by the Saha equation (ref. 8):

$$G \equiv K_{eq} \frac{kT}{P} = \frac{f_{eq}^2}{1 - f_{eq}^2} = \frac{BT^{2.5}}{P} e^{-E/kT} \tag{18}$$

where

$$B = \frac{\omega_i \omega_e}{\omega_a} k(2\pi m_e k)^{1.5}/h^3 \tag{19}$$

Equations (1) to (5) and (17) form a complete set of equations for the variables P, ρ, T, h, v, and f as functions of x. By specification of the stagnation conditions, these equations can be solved for the exhaust conditions for a given nozzle. One other relation of use in evaluating nozzle performance is the equation for the specific impulse at zero ambient pressure:

$$I_{sp} = \frac{F}{Wg} = g^{-1}\left(v_{ex} + \frac{P_{ex}}{\rho_{ex} v_{ex}}\right) \tag{20}$$

METHOD OF SOLUTION

In order to simplify the calculations, the pressure was specified as a suitable function of distance:

$$P = P_1 e^{-b(x/l)^2} \tag{21}$$

This removed the necessity of iterating to find the maximum flow rate for a nozzle of specified shape. The nozzle length l was taken as 20 centimeters, and the constant b was determined so that the inlet- to exhaust-pressure ratio was 1000. A typical nozzle profile resulting from this pressure function is shown in figure 1. The pressure and the degree of ionization were specified at the inlet, and the temperature was determined from the Saha equation (18). The inlet velocity was arbitrarily taken as one-tenth the speed of sound in the unionized gas at the inlet temperature (see table I). The calculations are insensitive to this choice.

With equation (18), the right side of equation (17) can be rewritten as

$$\frac{-\alpha(\rho/m_a)(f^2 - f_{eq}^2)}{(1 - f_{eq}^2)} \tag{22}$$

Near equilibrium

$$|f^2 - f_{eq}^2| \ll f_{eq}^2 \tag{23}$$

Evaluation by subtraction introduces insignificant figures. This difficulty was removed by linearizing the equations about the equilibrium values of the variables (see appendix B). The linearized equations were then solved, together with the equilibrium-flow equations (ref. 9). The nonlinear equations were used downstream of the point at which the departure of f from its equilibrium value amounted to 1 percent of the equilibrium value. All calculations were performed on an IBM 7090 computer.

DISCUSSION OF RESULTS

The calculations were performed for five cases. In four of these, lithium was used for the propellant as it combined the desirable qualities of low atomic weight and low ionization potential; two values of initial pressure and two values of initial degree of ionization were assumed. Cesium was used in the other case, for most of the experience with thermionic diodes to date has been with this material. The results of the computations are summarized in figure 2 and table I.

Figure 2 shows the degree of ionization f as a function of distance x/l for both exact and equilibrium solutions. The departure from equilibrium and subsequent asymptotic behavior of the exact solution is a feature of the well-known phenomenon of freezing (ref. 10). Figures 2(a) and (b) show that the onset of freezing occurs earlier in the cases with lower initial pressure. Throughout the range of these calculations, collisional recombination dominated the radiative process. Hence, the dependence of the collisional-recombination rate on the cube of the electron density, which is in turn proportional to the mass density, is responsible for this marked dependence of the freezing on initial pressure. The later onset of freezing for cesium is due to its much larger atomic weight and resulting slower passage through the nozzle.

Table I presents performance figures for the different cases and shows the superiority of the higher pressure lithium cases in both specific impulse and conversion efficiency. For example, in the case of lithium initially 60 percent ionized at 10 millimeters of mercury, a specific impulse of 960 seconds and 0.789 of the available thrust were obtained. The ratio $I_{sp}g/\sqrt{2h_0}$ is a measure of the conversion efficiency, because $\sqrt{2h_0}/g$ is the maximum attainable specific impulse. Since complete freezing occurred in the four lithium cases, any further expansion would yield only marginal increases in this ratio. The exhaust- to throat-area ratios, though quite large, are still manageable. The comparison of the exact and equilibrium values is somewhat misleading because of the larger exhaust areas for the equilibrium solutions. If, however, the exact solutions were continued to the same exit areas as the equilibrium solutions, the increase in specific impulse would be very small.

CONCLUDING REMARKS

Certain important effects have been neglected herein. Transport phenomena, such as ambipolar diffusion with subsequent surface recombination, may reduce the specific impulse. Moreover, there is some question about the temperature dependence of the collisional-recombination coefficient because experimental confirmation is lacking. Two conclusions based on this analysis seem justified, however. First, in order to recover most of the energy stored in the plasma, high initial pressures, 10 millimeters of mercury or higher, must be used. Second, if high degrees of ionization can be obtained with lithium at these pressures in a thermionic diode, then the system offers some promise as a thrust-producing engine.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, September 10, 1963

APPENDIX A

SYMBOLS

A^o, A^+	neutral atom, positive ion
a	cross-sectional area
B	constant in eq. (18)
b	constant in eq. (21)
E	ionization energy per atom
e^-	electron
F	thrust
f	degree of ionization, $n_e/(n_a + n_i)$
G	parameter in eq. (17)
g	gravitational constant
h	enthalpy per unit mass, also Planck's constant eq. (19)
I_{sp}	specific impulse
K	equilibrium constant
k	Boltzmann's constant
l	nozzle length
m	particle mass
n	particle number density
P	pressure
R	net reaction rate per unit volume per unit time
S	reaction rate constant
s'	constant entropy equation
T	absolute temperature
v	velocity
W	mass-flow rate

x distance from inlet

α recombination coefficient

δ departure from equilibrium

ρ mass density

ω statistical weight

Subscripts:

a neutral atom

c collisional

e electron

eq equilibrium

ex nozzle exit

i ion

ic collisional ionization

ir radiative ionization

r radiative

rc collisional recombination

rr radiative recombination

t throat

0 stagnation

1 initial state

Superscript:

$-$ equilibrium value

APPENDIX B

LINEARIZATION OF EQUATIONS

The nonlinear equations are as follows:

Energy:

$$h + \frac{1}{2} v^2 = h_0 \tag{B1}$$

Momentum:

$$\rho v \frac{dv}{dx} = - \frac{dP}{dx} \tag{B2}$$

Continuity:

$$\rho va = W \tag{B3}$$

Equations of state:

$$P = \rho \frac{k}{m_a} T(1 + f) \tag{B4}$$

$$h = 2.5 \frac{k}{m} T(1 + f) + \frac{E}{m_a} f \tag{B5}$$

Reaction rate:

$$v \frac{df}{dx} = -\alpha \frac{\rho}{m_a} \left[f^2 - G(1 - f^2)\right] \tag{B6}$$

$$G = \frac{BT^{2.5}}{P} e^{-E/kT} \tag{B7}$$

Near equilibrium, each of the variables can be represented by its value from the equilibrium solution plus a small perturbation. This gives

$$\rho = \bar{\rho} + \delta\rho \qquad v = \bar{v} + \delta v$$

$$T = \bar{T} + \delta T \qquad f = \bar{f} + \delta f$$

$$h = \bar{h} + \delta h \qquad a = \bar{a} + \delta a$$

$$P = \bar{P}$$

where the bars denote equilibrium values. There is no perturbation on P since it is specified. These quantities are substituted in equations (B1) to (B7), and products of small quantities are dropped. It may be noted that equations (B1) to (B5) are satisfied by the barred variables. The linearized versions of these equations are

$$\delta h + \bar{v}\,\delta v = 0 \tag{B8}$$

$$\frac{d}{dx}(\delta v) = \left(\frac{\delta v}{\bar{v}} + \frac{\delta\rho}{\bar{\rho}}\right)\frac{1}{\bar{\rho}\bar{v}}\frac{d\bar{P}}{dx} \tag{B9}$$

$$\frac{\delta\rho}{\bar{\rho}} + \frac{\delta v}{\bar{v}} + \frac{\delta a}{\bar{a}} = 0 \tag{B10}$$

$$\frac{\delta\rho}{\bar{\rho}} + \frac{\delta T}{\bar{T}} + \frac{\delta f}{1 + \bar{f}} = 0 \tag{B11}$$

$$\delta h = 2.5\,\frac{\bar{P}}{\bar{\rho}}\left(\frac{\delta T}{\bar{T}} + \frac{\delta f}{1 + \bar{f}}\right) + \frac{E}{m_a}\,\delta f \tag{B12}$$

From equation (B7),

$$G = G\left[1 + \left(2.5 + \frac{E}{k\bar{T}}\right)\frac{\delta T}{\bar{T}}\right] \tag{B13}$$

where, from equation (18),

$$\bar{G} = \frac{\bar{f}^2}{1 - \bar{f}^2} \tag{B14}$$

Finally, with (B13) and (B14), equation (B6) may be written as

$$\bar{v}\left(\frac{d\bar{f}}{dx} + \frac{d}{dx}\,\delta f\right) = -\bar{\alpha}\,\frac{\bar{\rho}}{m_a}\left[\frac{2\bar{f}}{1 - \bar{f}^2}\,\delta f - \bar{f}^2\left(2.5 + \frac{E}{k\bar{T}}\right)\frac{\delta T}{\bar{T}}\right]$$

Assuming $\frac{d}{dx}\,\delta f \ll \frac{d\bar{f}}{dx}$ gives

$$\left[\frac{2\bar{f}}{1 - \bar{f}^2}\,\delta f - \bar{f}^2\left(2.5 + \frac{E}{k\bar{T}}\right)\frac{\delta T}{\bar{T}}\right] = -\frac{\bar{v}m_a}{\bar{\alpha}\bar{\rho}}\,\frac{d\bar{f}}{dx} \tag{B15}$$

Equations (B8) to (B12) and (B15) form a complete set of equations for the variables $\delta\rho$, δT, δh, δv, δf, and δa once the equilibrium values are known.

The equilibrium-flow equations are (ref. 9)

Energy:

$$h + \frac{1}{2}v^2 = h_0 \tag{B16}$$

Entropy:

$$s' = s_0' \tag{B17}$$

Continuity:

$$\rho va = W \tag{B18}$$

Equations of state:

$$P = \rho \frac{k}{m_a} T(1 + f) \tag{B19}$$

$$h = 2.5 \frac{k}{m_a} T(1 + f) + \frac{E}{m_a} f \tag{B20}$$

$$s' = 2.5f + (1 + f)\ln \frac{BT^{2.5}}{P} + \ln \frac{1 + f}{1 - f} + f \ln \frac{1 - f^2}{f^2} \tag{B21}$$

Saha equation:

$$\ln \frac{BT^{2.5}}{P} = \frac{E}{kT} - \ln \frac{1 - f^2}{f^2} \tag{B22}$$

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TABLE I. - PERFORMANCE DATA FOR FIVE CASES

[Pressure ratio, 1000; nozzle length, 20 cm; initial velocity, $v_1 = 0.1\sqrt{5kT_1/3m_a}$.]

	Propellant				
	Lithium				Cesium
	Case				
	1	2	3	4	5
Initial pressure, P_1, mm Hg	10	10	1	1	10
Initial ionization, f_1	0.60	0.30	0.60	0.30	0.60
Initial temperature, T_1, $^{\circ}K$	5450	4830	4660	4200	4170
Specific impulse, I_{sp}, sec	960	800	836	699	199.
Conversion efficiency, $I_{sp}g/\sqrt{2h_0}$	0.789	0.861	0.706	0.776	0.834
Exit- to throat-area ratio, a_{ex}/a_t	50.4	44.5	40.2	37.3	64.3
Fraction of equilibrium specific impulse, $I_{sp}/I_{sp,eq}$	0.926	0.912	0.851	0.833	0.969
Equilibrium exit-area ratio, $a_{ex,eq}/a_{ex}$	1.81	1.82	2.42	2.44	1.38

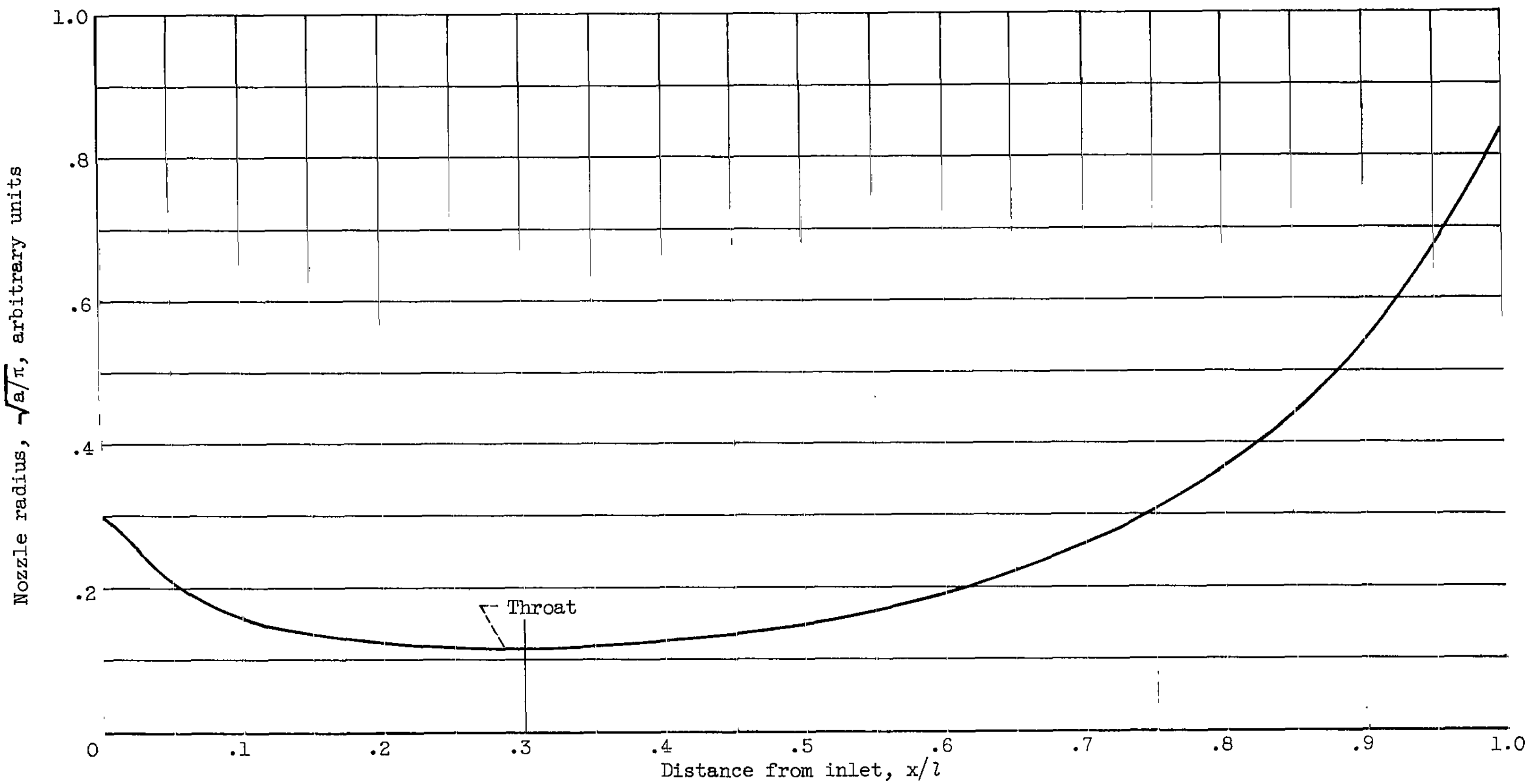
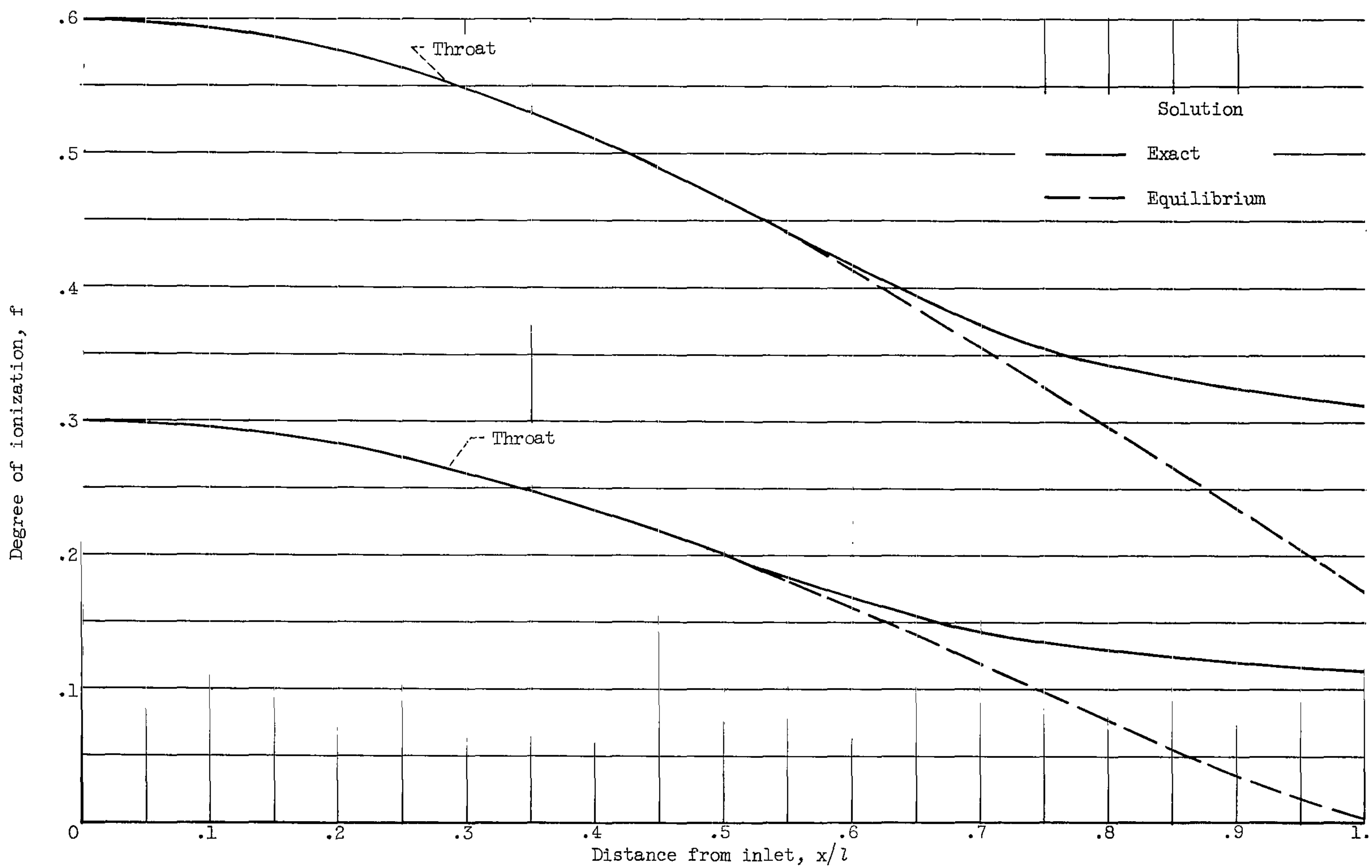
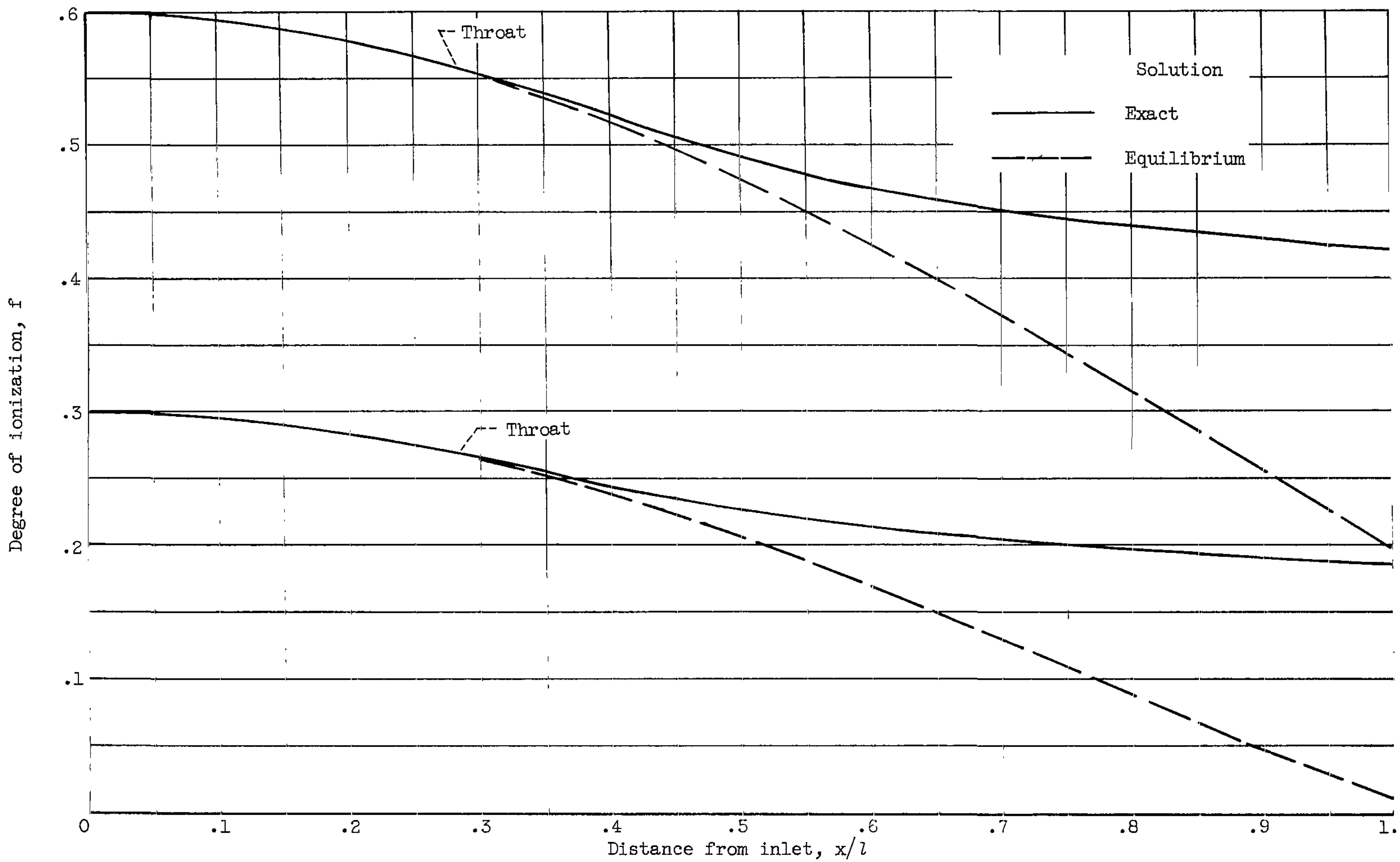


Figure 1. - Typical nozzle profile for lithium. Initial pressure, 10 millimeters of mercury; initial degree of ionization, 0.60.



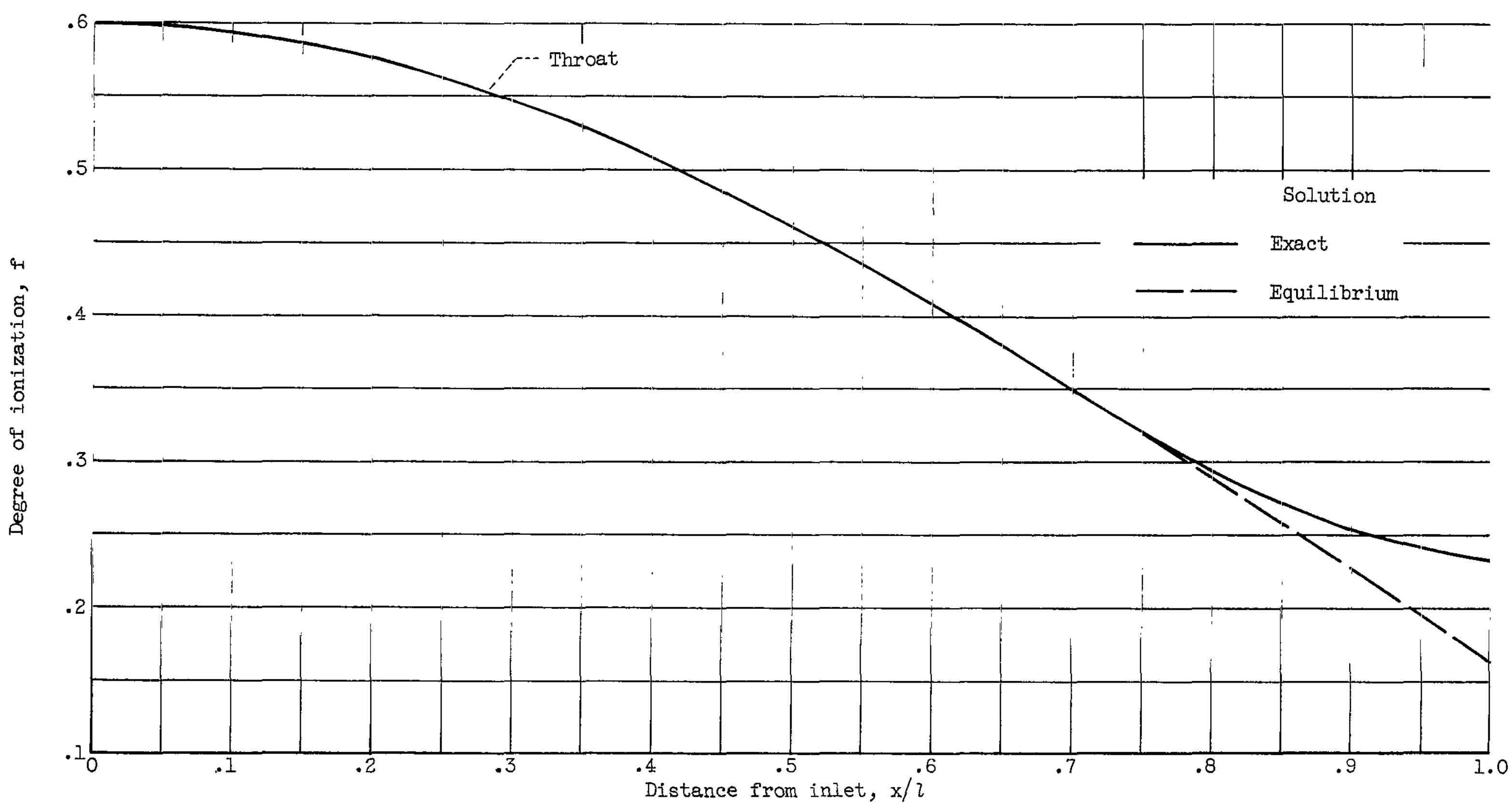
(a) Lithium propellant; initial pressure, 10 millimeters of mercury.

Figure 2. - Degree of ionization as function of distance.



(b) Lithium propellant; initial pressure, 1 millimeter of mercury.

Figure 2. - Continued. Degree of ionization as function of distance.



(c) Cesium propellant; initial pressure, 10 millimeters of mercury.

Figure 2. - Concluded. Degree of ionization as function of distance.

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